Nonlinear Logic (NLL) – Making Sense Out of Logical Self-Reference

Allan Goff*

Novatia Labs, Folsom, CA, 95630, AllanGoff@NovatiaLabs.com

The fundamental argument against any kind of faster-than-light (FTL) phenomenon is the issue of temporal paradox. Thanks to Special Relativity, FTL implies time travel and given time travel, the possibility of temporal paradox seems unavoidable. This is an argument from logic, not physics, yet upon closer examination it is discovered that our formal systems of logic studiously avoid self-reference in their foundational premises. This is doubly odd once it is realized that self-reference is unavoidable for any logical system of sufficient power, as demonstrated by Gödel's Incompleteness Theorem. This paper presents a brief overview of the major attempts to construct logical systems with self-reference built into their foundations. It then covers in some detail Nonlinear Logic (NLL) the self-referential logical system developed by Novatia Labs as part of their research project into the potential for quantum systems to support superluminal communications. Like most formal systems of logic, statements in NLL consist of typographical strings. The major addition to the symbol set is the label operator so that self-reference can be indicated. There is also a symbolic representation of statements in NLL derived from digital circuits that is isomorphic with the typographical representation. The digital circuit formalism is useful for gaining insight into what self-reference brings to systems of logic. It also suggests that sequential circuits could be designed with greater efficiency using NLL than standard Boolean logic. As with other self-referential logics, NLL supports more truthvalues than just true and false. These imaginary or complex truthvalues can be represented with an excursion into time (NLL) or into space (Kaufman's Knot Theory). NLL also supports a graphical representation of truthvalue called attractor structures. This representation has its roots in the "attractors" of chaos theory, which has its basis in nonlinear mathematics. The resulting system of logic is closed, complete, and demonstrates that sense can be made of logical paradox. In so doing, it weakens the argument that temporal paradox prohibits time travel and in turn FTL, hinting at possible loopholes in physics for these phenomena.

Nomenclature

A, B, C	=	Propositions
T, **F**	=	Linear truthvalues
i, **j**	=	Imaginary truthvalues
~ ≈ ^ ⊕ ≡ ⊃ v	=	Logic operators
:	=	Label operator (for reentrance and self-reference)

I. Introduction

THIS paper is the first of eight integrated papers[1,2,3,4,5,6,7,8] to explore the potential of quantum nonlocality to support superluminal signaling; i.e., communicating at faster-than-light (FTL) speeds[9]. Spacelike causality raises a number of issues that must be addressed if nature is to permit any kind of FTL phenomenon. These include consistency with Special Relativity, a broadened formulation of causality, and either resolution or avoidance of temporal paradox[10]. These issues are addressed with increasing sophistication through the series of eight papers.

This paper covers the essentials of modern symbolic logic, presents a survey of the leading formal logics that can indicate and evaluate self-referential forms and briefly introduces Novatia Labs' Non Linear Logic (NLL). NLL is then used to demonstrate that quantization is one way to avoid infinities, that acausality is to be expected in physics,

* President, Novatia Labs, 9580 Oak Ave. Parkway, #7-110, AIAA Member.

that complex truthvalues may form an alternative mathematical foundation for quantum mechanics and that the arguments of temporal paradox and temporal indeterminacy against FTL are not as powerful as they first appear.

II. Background

This section introduces the propositional calculus, a typographical formal system of logic, describes Gödel's Incompleteness Theorem demonstrating that logical self-reference is unavoidable in a useful system of logic, and presents a brief survey of the leading self-referential logics.

A. The Propositional Calculus

Also called symbolic logic, the propositional calculus[11] is a typographical formal system that deals with statements that can be either true or false. The primitives are the two truthvalues, **T** & **F**, a basic set of binary and unary operators such as and "^", implies "⊃", and not "~", and letters to represent expressions. An initial set of true statements are given as axioms which can be converted into other true statements using rules of transformations that preserve truthvalues. These transformation rules are intended to capture logical thought. A sequence of transformations constitutes a proof; where the last statement in the sequence is the theorem (statement) that is proven. A statement that is true regardless of the value of the expressions that make it up is called a tautology. A formal system is consistent if only tautologies can be proven; it is complete if there is a proof for every one.

B. Russell's Paradox

Set theory is a foundational element of modern mathematics, but it is not without its problems. Consider that some sets cannot be members of themselves, such as the set of primes, but other sets can be members of themselves, such as the set of all sets. Clearly, a set is either a member of itself or not, so Russell simply considered the set all sets that are not members of themselves. If it is a member it should not be, but if is not, it should be, a logical contradiction. Russell's paradox[12] raises difficult questions about the foundation of set theory, and in turn mathematics itself.

C. Gödel's Incompleteness Theorem

Russell's paradox enticed Gödel to consider the ramifications of self-reference on formal systems in general. He did this by creating a clever mapping between the statements of a formal system and the numbers that the formal system could represent. Through this dual use of the symbols (the use/mention distinction), he showed that formal systems of any reasonable power could refer to themselves. In so doing, he effectively broadened Russell's paradox, showed that formal systems were either inconsistent or incomplete, that there were statements that could be seen to be true from outside the system, but could not be proven true within the system, and so showed that truth was a "higher" concept than provability.[13]

D. Some Self-Referential Logics

The holes in logic demonstrated by Russell and Gödel require self-reference. Russell advanced the "Theory of Types" to prevent such self-reference, and no holes are known of that don't ultimately depend on self-reference. This has led some researchers to the conclusion that self-reference must be dealt with from the beginning, at the axiomatic level.[14]

1. Laws of Form

The first effort to deal with self-referential logics was "Laws of Form" by G. Spencer-Brown where he derived Boolean logic from first principles and introduced a notation that in general use is not typographic but can be converted to one[15]. In this small book, Spencer-Brown develops his system of logic until it can represent self-referential logical forms through an excursion to infinity. He was the first to propose imaginary truthvalues as the resolution to logical paradox, but did not formally introduce them into his system.

2. Diamond Logic

Hellerstein developed a system of self-referential logic "Diamond Logic" utilizing two imaginary truthvalues, **i** and **j**[16], as did Kauffman[17] but it is not clear how to generalize these so they can handle complex truthvalues.

3. Knot Theory

Louis Kaufman of the University of Chicago has taken a slightly different attack creating a system of self-referential logic based on the excursion into space.[18] He is probably the current leading researcher in this area.[19]

A	B	Contra-diction	Tautology
F	**F**	**F**	**T**
F	**T**	**F**	**T**
T	**F**	**F**	**T**
T	**T**	**F**	**T**

A	B	~(A∨B)	A∨B
F	**F**	**T**	**F**
F	**T**	**F**	**T**
T	**F**	**F**	**T**
T	**T**	**F**	**T**

A	B	~(B⊃A)	B⊃A
F	**F**	**F**	**T**
F	**T**	**T**	**F**
T	**F**	**F**	**T**
T	**T**	**F**	**T**

A	B	~(A⊃B)	A⊃B
F	**F**	**F**	**T**
F	**T**	**F**	**T**
T	**F**	**T**	**F**
T	**T**	**F**	**T**

A	B	A∧B	~(A∧B)
F	**F**	**F**	**T**
F	**T**	**F**	**T**
T	**F**	**F**	**T**
T	**T**	**T**	**F**

A	B	~A	A
F	**F**	**T**	**F**
F	**T**	**T**	**F**
T	**F**	**F**	**T**
T	**T**	**F**	**T**

A	B	~B	B
F	**F**	**T**	**F**
F	**T**	**F**	**T**
T	**F**	**T**	**F**
T	**T**	**F**	**T**

A	B	A≡B	A⊕B
F	**F**	**T**	**F**
F	**T**	**F**	**T**
T	**F**	**F**	**T**
T	**T**	**T**	**F**

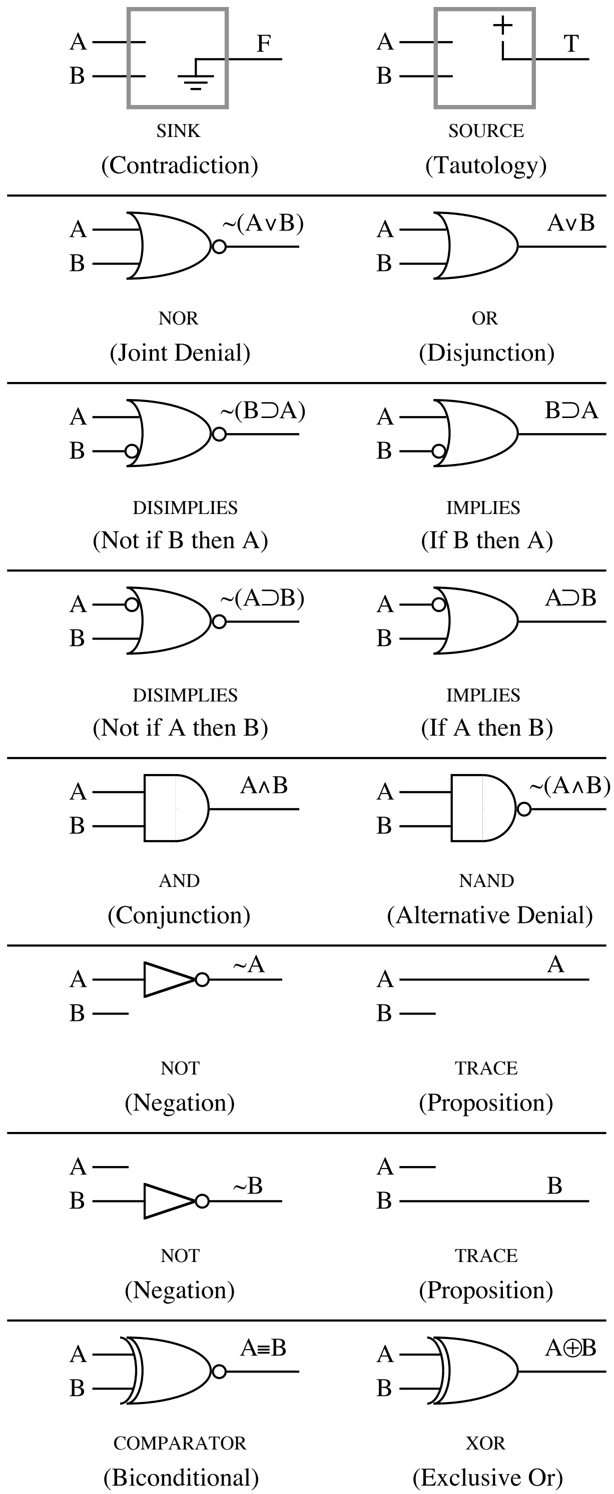

Figure 1 - Complete set of binary truth tables, gates, and operators.

III. Non Linear Logic (NLL)

This section presents the highlights of Novatia Labs' Non Linear Logic (NLL). In the short space provided by this venue, this is a necessarily very brief look at a deep and complex subject. For a popular account of nonlinear systems see Gleick.[20] The purpose of this description is to provide the reader with some basis to believe that the temporal paradox argument against FTL is not as strong as it first appears.

NLL is a dual notation system of self-referential logic. One notation is typographic; the other is derived from digital circuit schematics and is therefore pictorial. They are, however, isomorphic with each other. NLL generalizes the propositional calculus; it introduces the label operator, which makes it possible to indicate self-reference, and it expands the basic set of binary operators to include all possible 2-input truth tables. From the propositional calculus, the excess of binary operators seems verbose, but in a self-referential setting, it turns out to be essential. Unlike in symbolic logic where statements can be reduced to expressions utilizing only one of either two binary operators (NOR or NAND), self-referential forms cannot be so simplified. Indeed, it is necessary to allow a construct that allows for the explicit construction of n-input operators.

The complete set of binary truth tables, gates, and operators are shown in Figure 1.

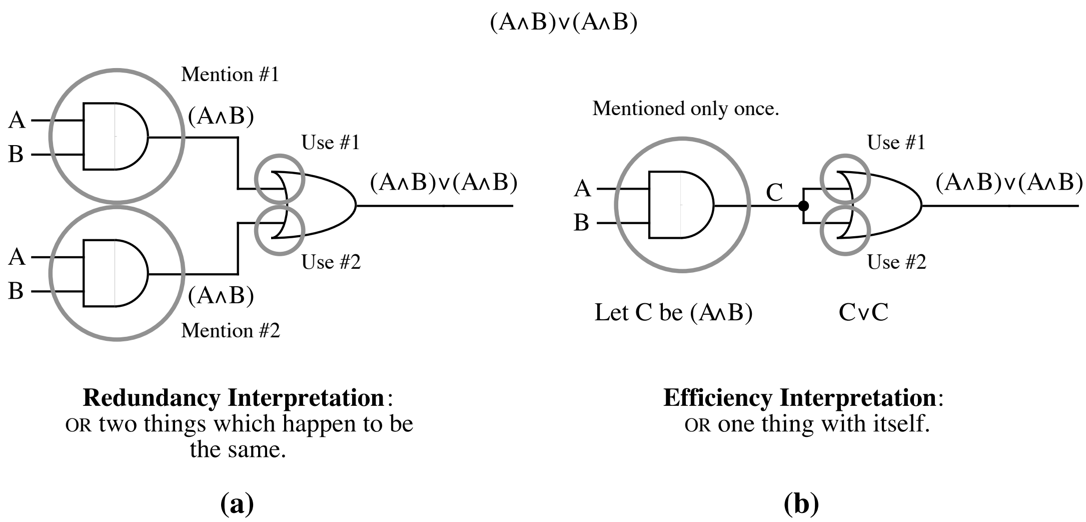

Figure 2 – Example of the use/mention distinction.

A. Digital Circuit Foundation

The digital circuit notation reveals a hidden assumption in the propositional calculus; the degeneracy between use and mention that Gödel uses in his Incompleteness Theorem. An example is shown in Figure 2, where two different circuits would have identical representations in the propositional calculus. In the digit circuit notation, it is trivially easy to indicate self-reference via a feedback loop. Figure 3 shows an example. For this to have unambiguous meaning the gates need to be clocked. Therefore, the final addition to this notation is a way to indicate when a group of gates should be individually unclocked, but ganged together into a clocked circuit. The clock is assumed universal.

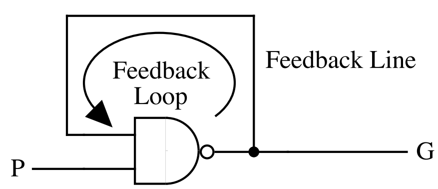

Figure 3 – Self-reference as feedback.

B. Typographical Representation

The digital circuit notation reveals the limitations of the propositional calculus. To overcome theses limitations, the typographical notation of NLL adds a label operator and square braces. The label operator breaks the use/mention degeneracy and permits the indication of self-reference, while the square braces indicate when a circuit is to be considered constructed out of unclocked gates, but clocked as a unit. This completes the isomorphism between the two notations and allows an unambiguous evaluation of all logical forms, in particular, the self-referential ones.

C. Attractor Structures

Because of the clocked gate requirement, a self-referential form typically has one or more repeating sequences of true and false values. These sequences are best represented graphically as attractor structures. An attractor structure consists of nodes, one for each possible state of the circuit, and connecting lines that show the successor node of every possible state. A single attractor consists of a repeating sequence of truthvalues shown as a circle with nodes on it. The convention is to traverse the circle clockwise. There may also be predecessor nodes that are not on the repeating cycle called stems. The stems represent transient values. The repeating sequences represent paradoxical values, and the different attractors within an attractor structure represent the indeterminacy of the values. A form with a conventional linear truthvalue will consist of a single attractor with a cycle consisting of a single node. Figure 4 shows examples of several attractors (a-c) plus the attractor structures and circuits for three simple self-referential forms (d-f) and just the attractors for a simple RS flip-flop (g).

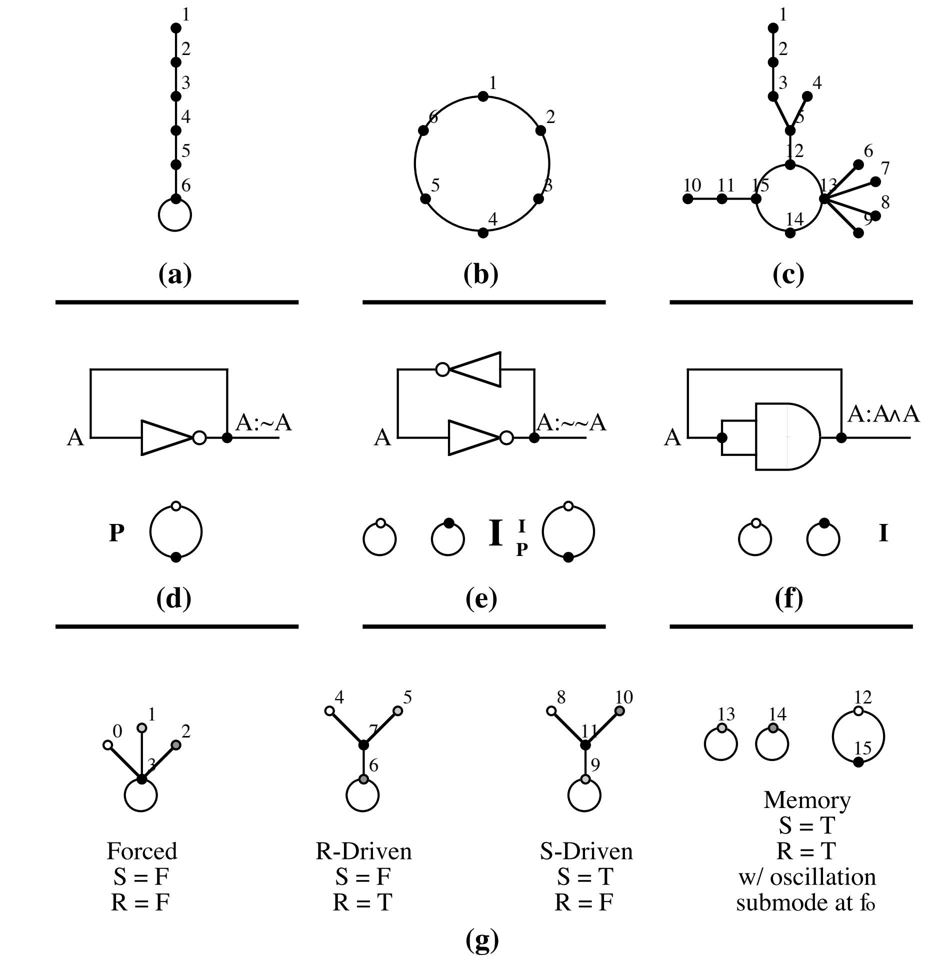

Figure 4 – Examples of attractor structures (a-c). Simple self-referential forms and their attractor structures (d-f). The attractor structures for an RS flip-flop (g).

IV. Representations and Evaluations

The development so far is internally consistent and permits an interpretation of every form. For a similar line of development, see Shoup,[21] in particular his "square root of a NOT gate". However, more work is required before we can tie these forms back to the conventional logical concepts of value and evaluation.

A. The Paradox Frequency

Consider again the Liar's Paradox (Figure 4a). This simple NOT gate feeding back on itself represents the most basic logical paradox. The clocked gate metaphor (and the attractor structure view) imply that it should oscillate, but at what frequency? Note that a real TTL NOT gate with its output tied back to its input will not oscillate with either a capacitor or inductor element since electronically it is in fact just a very high gain analog amplifier. In particular, would two different paradoxes oscillate at the same frequency?

NLL answers this question with the following line of reasoning. Assume a maximum but finite frequency. Then consider the circuit shown in Figure 5 where two different paradoxes feed an XOR gate. If they oscillate at the same frequency, but with a phase difference of 90° then the output of the XOR gate has twice the frequency violating our assumption of a maximum frequency. There are only two solutions; let the frequency go to infinity or quantize the phase. An infinite frequency presents severe interpretational challenges, so we opt for the latter solution and note that the clocked gate metaphor automatically quantizes phase. The maximum frequency is thus also the paradox frequency.

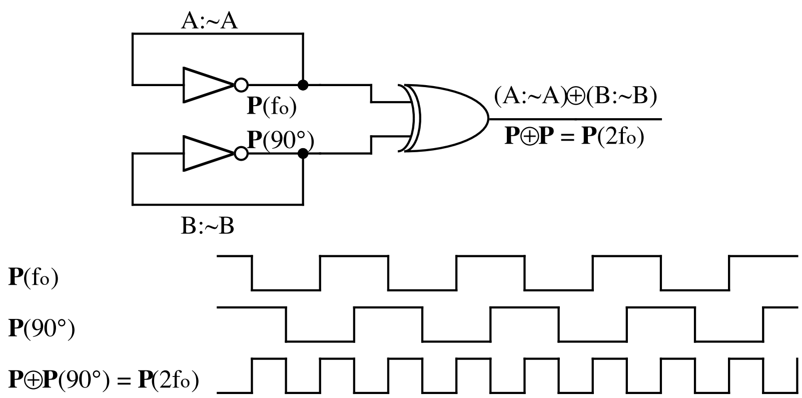

Figure 5 – The paradox frequency. To map paradox to an oscillation of finite frequency, both phase and frequency must be quantized.

B. The Clocked Gate Metaphor

The clocked gate metaphor simply assumes a universal clock. Every gate (or compound gate) receives this clock signal. All outputs transition to be in accord with their inputs on the same phase of the clock. Transmission times are ignored under the simplifying assumption that they are all shorter than the cycle duration of the clock. This permits an unambiguous and deterministic attractor structure for every nonlinear logical form and circuit.

C. A Digital Oscillator

Figure 6 shows an all-digital oscillator with its attractor structure under the clocked gate metaphor. We actually constructed this circuit with real TTL gates and if the two flip-flops were on different chips manufactured by different companies it would oscillate, erratically, but it would not settle down. This circuit produces an attractor structure with two attractors, one at the paradox frequency and the other at one quarter of the paradox frequency but with a 5/8th duty cycle, so it is indeterminate in two different paradoxes.

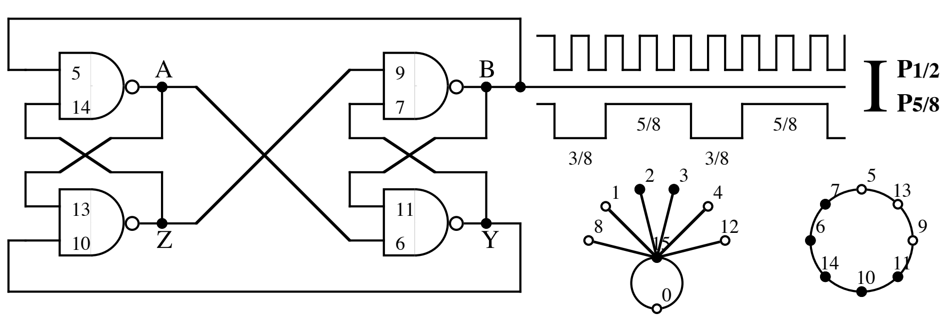

Figure 6 – A digital oscillator indeterminate between two different paradoxes.

V. Theorems & Proofs

The fundamental basis for the concept of a proof is the idea that form and value are separate concepts, that it is possible to change one without changing the other. Such transformations generally fall into two categories; changing the form of an expression without changing its value, or changing two equivalent expressions in such a way that their values change in the same way and thus remain equivalent.

A. Ganged Gates

In general, we are interested in forms with N clocked gates. Each of the clocked gates may be a simple unary or binary gate, or it may be a combination of such gates all clocked as a unit. We call this a ganged gate. Ganged gates permit the construction of circuits that can implement arbitrary truth tables with any number of inputs. Any self-referential form can be expressed as N clocked ganged gates each with N inputs, one from each of the ganged gates. There are $n=2^N$ possible truth tables for an N-input ganged gate, and therefore n^n possible circuits.

There is the same number of possible attractor structures in a one-to-one correspondence. For an N gate form, there are 2^N possible nodes, and since any node can have any node as a successor, there are n^n possible attractor structures. If all predecessors are unique, then the attractors are pure cycles (stemless) and there are only n! of them.

(A:[A⊃B∧C]),(B:[A⊃B∨C]),(C:[A⊃B~∧C])

x	A	B	C	A:[A⊃B∧C]	B:[A⊃B∨C]	C:[A⊃B~∧C]	Sx
0	**F**	**F**	**F**	**F**	**T**	**T**	3
1	**F**	**F**	**T**	**T**	**T**	**F**	6
2	**F**	**T**	**F**	**F**	**T**	**T**	3
3	**F**	**T**	**T**	**T**	**T**	**F**	6
4	**T**	**F**	**F**	**F**	**F**	**T**	1
5	**T**	**F**	**T**	**F**	**T**	**T**	3
6	**T**	**T**	**F**	**F**	**T**	**T**	3
7	**T**	**T**	**T**	**T**	**T**	**F**	6

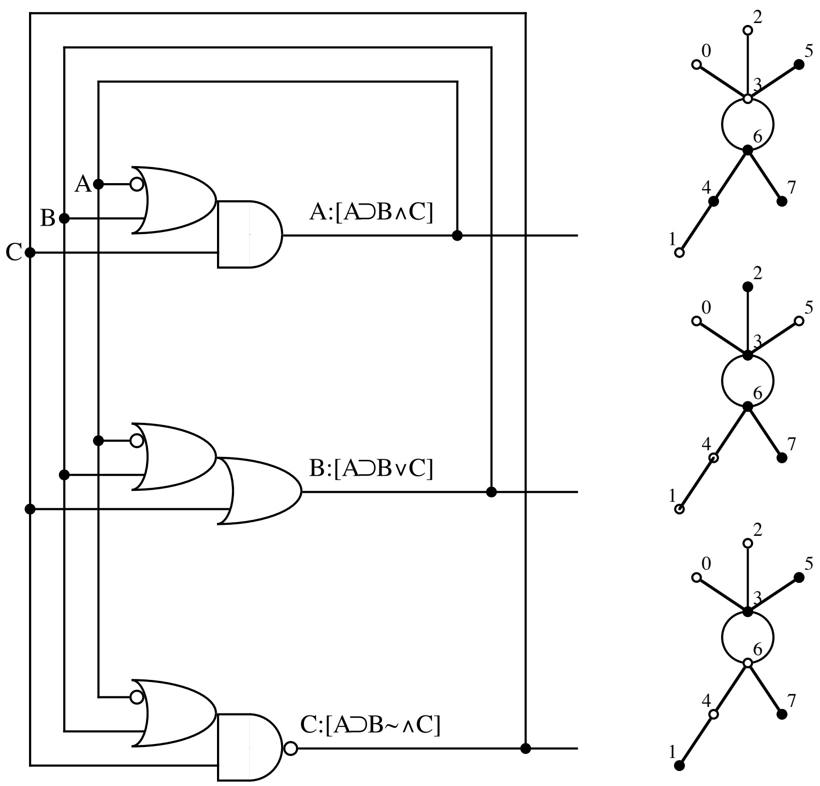

Figure 7 – Ganged gates. All self-referential forms come down to n, n-input gates where all outputs feed all inputs.

Figure 7 shows an example of a 3-input self-referential form that utilizes ganged gates. The form is shown in five different equivalent representations; a statement in the nonlinear logic, a truth table, a successor vector (S_x), a digital circuit, and three instances of the attractor structure with different black and white coding.

The nonlinear logic statement uses the comma separator and label operator to indicate the sub expression for each of the three ganged-gates. The truth table reveals the successor of each of the 8 possible states of the form converting 'x' into 'Sx', called the successor vector. The digital circuit uses basic binary gates to compose the three input ganged gates. Binary gates that touch each other are clocked as a group, not separately. In more complicated forms the binary gates composing a ganged gate may be outlined with a dotted box, the pictorial equivalent of the label operator. The individual binary gates in a ganged-gate are not clocked, only the output from the final gate is clocked. Three outputs imply 8 nodes. Every ganged gate receives as input the output of every ganged gate including itself. This is a general architecture of nonlinear forms. The attractor structure is constructed directly from the successor vector. It is shown thrice, once for each output, and consists of a single 2-banger attractor in the attractor structure with a maximum transient depth of 3. The eight nodes are either black or white indicating whether that output is true or false respectively. With this symbology it is easy to deduce that two of the three gates (A & C) oscillate at the paradox frequency but out of phase, while the other (B) is always true, a tautology. Even though A and C are both paradoxes, because they are out of phase, A OR C is tautological.

B. Smullyan's Sanity Land

In the 1990's Smullyan created a series of self-referential logic puzzles based in Sanity Land where there is a tight formal relationship between sanity (S), belief (B), and matters of fact (M).[22] There are only two types of people in Sanity Land, the sane and the insane. The sane believe matters of fact while the insane do not. Similarly, the insane do believe in false matters of fact while the sane do not. In NLL, this basic relationship is represented as a flip-flop composed of two COMPARATOR (NOT XOR) gates with the free inputs tied together. The tied input is the matter of fact, the two outputs sanity and belief.

We'll only consider one of Smullyan's puzzles, his first one, "What is the situation for a patient who believes that not both he and his doctor are sane?" The matters of fact are the doctor's sanity (D), that not both are sane (M) the patient's belief (B) which by the puzzle is true, and his sanity (S). The NLL expression, circuit, and attractor structure are shown in Figure 8. The node number is a binary interpretation of DMBS (8-4-2-1). For example, node 5 (0101) implies an insane doctor, not both sane, patient insane but believes that not both he and the doctor are sane. Node 5 has node 6 as a successor, (and vice versa) so this combination is paradoxical.

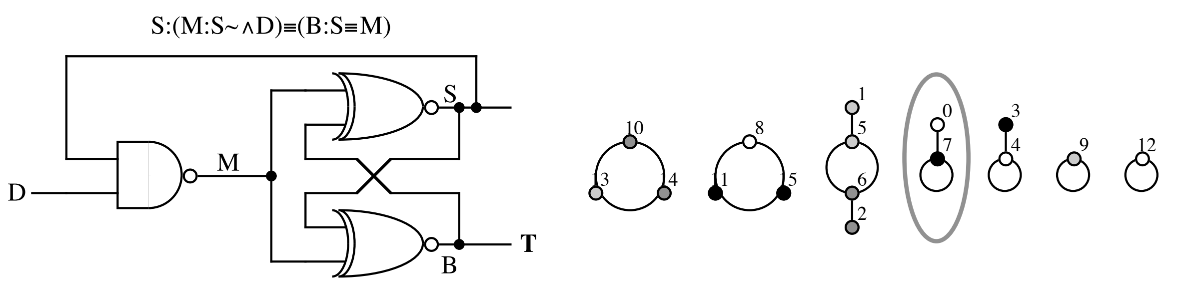

Figure 8 – One of Smullyan's Sanity Land puzzles.

If the doctor is sane, the patient's belief is either paradoxical or false, not true as given. The only attractor that meets the criteria is the singlet with node 7. The binary representation of node seven reveals that the doctor is insane, the patient sane, not both are sane, and the patient believes that. This is precisely the answer that Smullyan gives. However, instead of reasoning to it, NLL permits the answer to be computed.

C. Derivation of the 'D' Flip-Flop Tautology

In linear logics, proofs cannot be done with self-referential elements. NLL has been developed far enough that some simple proofs can be done with self-referential forms. A complete axiomatic foundation is incomplete largely because the concept of equality is degenerate in linear system and needs a formal taxonomy in self-referential logics. Each taxonomy permits its own set of truthvalue-preserving transformations and these have not been worked out. Despite this current shortcoming, some proofs are possible even at this level of development. Equation 2 shows the derivation of the 'D' flip-flop tautology.

$$
\begin{array}{rcl}
\sim D \oplus D & . & \text{axiom} \\
\sim D \oplus \sim\sim D & . & A = \sim\sim A \\
(D \sim \wedge D) \oplus \sim\sim D & . & \sim A = A \sim \wedge A \\
(Q_1 : D \sim \wedge D) \oplus \sim\sim D & . & \text{Label Rule} \\
(Q_1 : D \sim \wedge \sim\sim D) \oplus \sim\sim D & . & A = \sim\sim A \\
(Q_1 : D \sim \wedge T_1) \oplus (T_1 : \sim\sim D) & . & \text{Efficiency Rule} \\
(Q_1 : D \sim \wedge T_1) \oplus (T_1 : \sim D \sim \wedge \sim D) & . & \sim A = A \sim \wedge A \\
(Q_1 : D \sim \wedge T_1) \oplus (T_1 : \sim D \sim \wedge (\sim\sim D \sim \wedge \sim\sim D)) & . & A = \sim A \sim \wedge \sim A \\
(Q_1 : D \sim \wedge T_1) \oplus (T_1 : \sim D \sim \wedge (\sim\sim D \sim \wedge (\sim\sim\sim D \sim \wedge \sim\sim\sim D))) & . & A = \sim A \sim \wedge \sim A \\
(Q_1 : D \sim \wedge T_1) \oplus (T_1 : \sim D \sim \wedge (D \sim \wedge (\sim D \sim \wedge \sim D))) & . & A = \sim\sim A \text{ (thrice)} \\
(Q_1 : D \sim \wedge T_1) \oplus (T_1 : \sim D \sim \wedge (Q_2 : D \sim \wedge (T_2 : \sim D \sim \wedge \sim D))) & . & \text{Label Rule} \\
(Q : D \sim \wedge T) \oplus (T : \sim D \sim \wedge Q) & . & \text{Recursion Rule}
\end{array}
\qquad (1)
$$

VI. Implications for Physics

Self-reference presents two conundrums for linear logics, paradoxes and circular arguments. NLL shows that both can be made logical and internally consistent. If self-reference is the key to the quantum measurement problem, then NLL provides a beginning mathematical and conceptual foundation for the measurement process.

A. Acausality

The first implication of self-reference is the separation of form and content.[23] A flip-flop is a controllable memory element that is logically self-referential. The content depends on the history of the circuit, not just on its logical form. In NLL, the content of a self-referential form is acausal; it may take on either linear logical value for an indeterminate form, and either phase for a paradoxical form. One of the great mysteries of quantum physics is, where does the acausality come from? Once content has become separated from form, the causal chain is broken and it becomes possible to have acausality.

B. Complex Truthvalues

Matrix math was put to use early in the history of quantum mechanics because it could be used to compute answers that agreed with experiment. However, matrices may be overkill, because of the huge degeneracy of values that correspond to the same physical state. Complex truthvalues are a potentially stronger mathematical basis for quantum mechanics because they avoid this degeneracy. Much work remains to be done in this area.

C. Temporal Paradox and Temporal Indeterminacy

The most exciting implication of NLL is that it shows that paradox does not have to break logic. If paradox can be permitted within the right logical framework, then perhaps it can also be permitted within the right physical framework. If so, then temporal paradox may have a viable physical interpretation that does not a priori make time travel impossible. If time travel is possible, then so is FTL, indeed, if and only if. This is in fact the primary motivation for developing NLL.

VII. Conclusion

NLL demonstrates that sense can be made of logical self-reference. While it uses an excursion into time to evaluate paradoxical truthvalues instead of complex truthvalues, it does demonstrate the separation of form and content implied by self-reference and supports a consistent methodology in dealing with both circular arguments and paradoxes. It is isomorphic with digital circuits and offers an approach to circuit design that may be more optimal than current approaches. It resolves Russell's paradox and suggests that Gödel's Incompleteness Theorem is really an argument for complex truthvalues, not an argument that formal systems are incomplete. These results are consistent with other self-referential logics. Key results are the number of possible self-referential forms, the use of attractor structures as an evaluation method, the quantification of phase, a demonstration of logical analysis of a self-referential form with a linear truthvalue, and one example of a proof that derives a tautological self-referential form. The key motivation in developing NLL was to demonstrate that logical paradox is not nonsensical, therefore weakening the argument that temporal paradox prohibits FTL.

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