Derivation of the Symmetric Spacetime Interval – A Formalism for Relativistically Consistent Wave Function Collapse

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It has previously been shown that collapse of the quantum wave function can be made consistent with relativity by specifying collapse to occur along special spacelike spacetime intervals. The special case of two entangled particles has been considered in some detail. When the entangled particles have the same mass, the world line of their center of mass bisects the spacetime interval along which collapse occurs, so we have taken to calling them symmetric spacetime intervals. In previous work, symmetric spacetime intervals were taken as a hypothesis. In this paper, they are derived and the concept is shown to apply equally well to entangled photons at light speed and to matter particles at sublight speeds. The derivation shows that the slope of the symmetric spacetime interval is numerically equal to the velocity of the center of mass of the entangled pair as previously hypothesized.

Nomenclature

x = normalized distance in light seconds (ls)
t = normalized time in seconds (s)
γ = Lorentz factor
s = spacetime interval
u = normalized velocity (ls/s)
c = speed of light (1.0 ls/s)
d = separation between pulses
L = separation between detectors
ν = frequency
n = number of pulses
β = slope of the symmetric interval (s/ls)

I. Introduction

THIS paper is the third of eight integrated papers[1,2,3,4,5,6,7,8] to explore the potential of quantum nonlocality to support superluminal signaling; i.e., communicating at faster-than-light (FTL) speeds[9]. Spacelike causality raises a number of issues that must be addressed if nature is to permit any kind of FTL phenomenon. These include consistency with Special Relativity, a broadened formulation of causality, and either resolution or avoidance of temporal paradox[10]. These issues are addressed with increasing sophistication through the series of eight papers.

This paper derives the mathematical form of the symmetric interval for a pair of entangled photons. As hypothesized, it is shown that the slope of the symmetric interval is equal to the velocity of the center of mass of the entangled particles. While derived for the special case of photons, the hypothesis is generalized to apply to matter particles as well and it is shown that this is at least internally consistent. Given the experimental particulars from any frame of reference, all relativistic observers can agree on the symmetric interval and therefore agree on which observation collapsed the wave function, even though in general this collapse is seen to proceed forward in time from some frames and backwards in time for other frames.

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II. Background

The previous paper covered the relevant fundamentals of relativity and quantum mechanics, introduced the 12 Light Second thought experiment, and presented the hypothesis it leads to, i.e., that wave function collapse occurs along symmetric spacetime intervals.

III. Conceptual Development

In relativity, we specify the evolving state of an object as a world line, presumed to be infinitely thin.[11] Because of the uncertainty principle, however, the world line for a quantum object is thick.[12] The thickness of the world line indicates the uncertainty in the object's position and the rate at which its thickness increases indicates the uncertainty in its momentum.

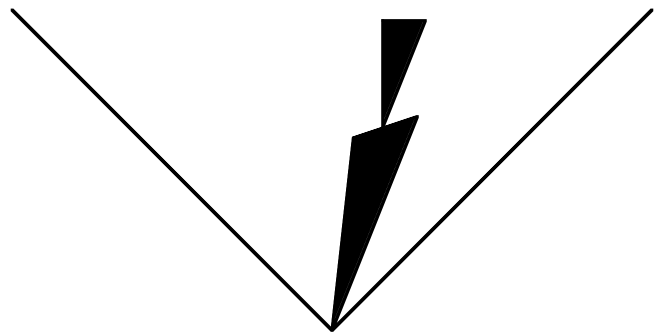

Figure 1: The world line of a quantum object before and after a position measurement.

Now consider a position measurement on this quantum object. Its world line suffers a discontinuous reduction in its thickness to some random point (Figure 1). The world line continues to broaden from that point onward, which visually looks like two narrow upside down triangles with the tip of the upper one touching the base of the lower one. In principle, the upper edge of the lower triangle could have any shape that did not include a timelike slope, but for the moment, assume it is just a straight line. This straight line is a spacelike spacetime interval and will Lorentz transform like any other spacetime interval. From all reference frames except the simultaneous one, collapse proceeds forward in time in one direction from the measured position and backward in time in the other direction. If its slope can be deduced, then all relativistic observers can simply Lorentz transform it from their frame to any other frame, and we will have achieved our objective; relativistically consistent collapse of the wave function.

IV. Derivation

Figure 2 shows the physical situation we are going to analyze. Two entangled photons were emitted from a central source and sent in opposite directions toward two detectors. The only variables in this setup are the frequencies of the photons (ν_1 & ν_2) and the distances from the source to each detector (*A* & *B*). We assume that data will be gathered from an arbitrary reference frame whose origin is at the creation of the entangled photons. There will be another frame of interest, U, the frame where the frequencies of the two photons are the same. The photon to the left will be photon 1, which is observed at detector A; and the photon to the right will be photon 2, which is observed at detector B. The world lines of the photons lie on the light cone. The world lines of the detectors are vertical, the world line of the center of mass of the photons leans toward the right so the photon on the right has the higher frequency. Also shown is the plane of simultaneity for the frame of reference where the two photons are detected simultaneously and the symmetric spacetime interval (bold). Dots indicate the four spacetime events of interest; creation (0, 0), the detection of the encounter particle (7, 7), the location of the responder particle at the collapse of the wave function (-3.5, 3.5), and the detection of the responder particle (-5, 5). Note that the slope of the symmetric interval is equal to the velocity of the center of mass of the entangled particles, and that the collapse of the wave function proceeds backwards in time in this frame of reference.

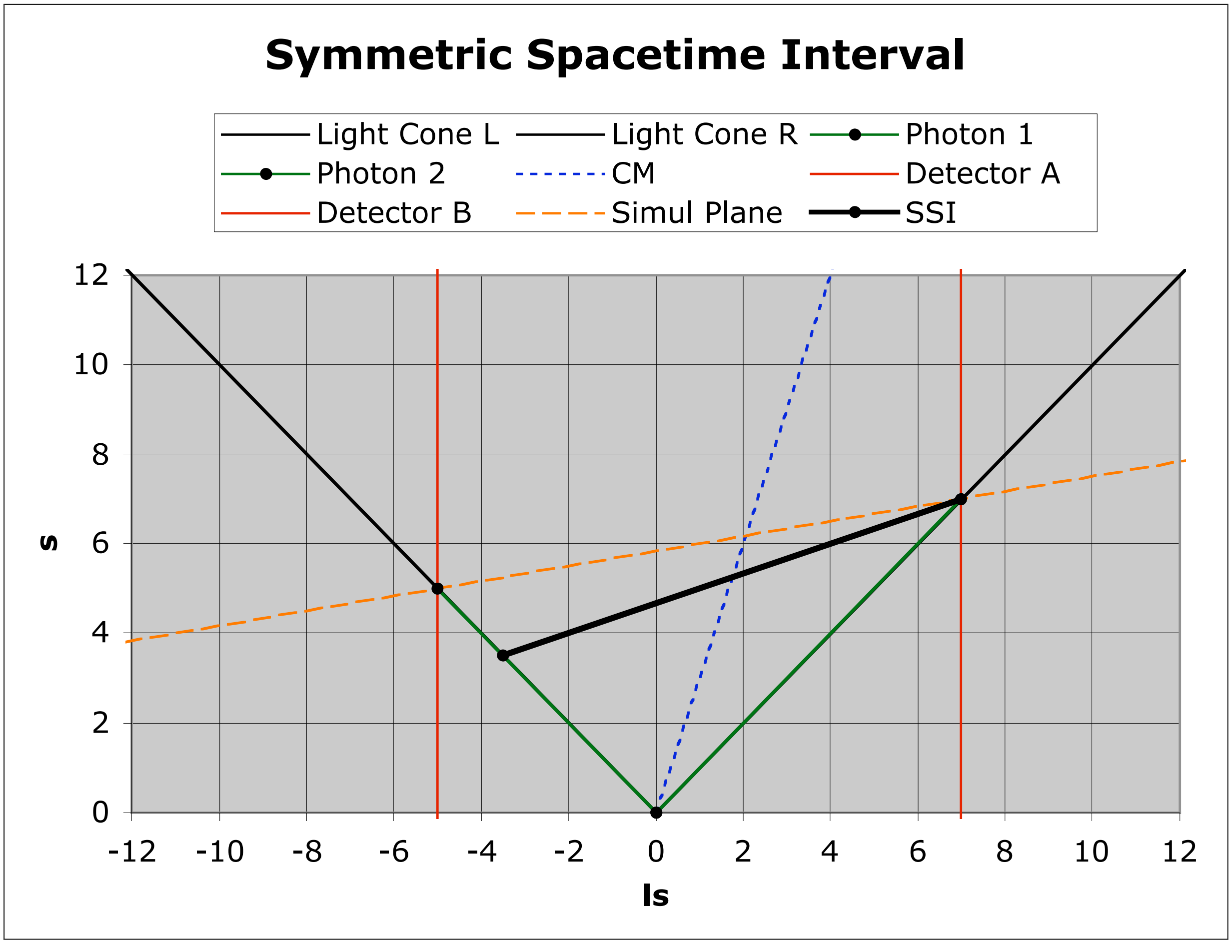

Figure 2: Spacetime diagram of two entangled photons (as seen from the lab frame) showing the symmetric spacetime interval connecting them at the moment of collapse.

Parameters of importance are the velocity, u, of the frame where the photon frequencies are the same, and the slope of the symmetric interval, β. It is also important to distinguish the photon whose measurement is the cause of the wave function collapse (the encounter particle, parameters subscripted with an 'e') from the other photon, the one that is measured after collapse of the wave function occurs (the responder particle, parameters subscripted with an 'r'). Finally, the parameters associated with collapse of the wave function will be subscripted with a 'c'. There are four spacetime events of interest,

1. Creation of the entangled photons
2. Detection of the encounter particle
3. Collapse of the wave function
4. Detection of the responder particle

The advantage of considering photons is that the slope of their world lines won't change with a change in reference frame. The only changes will be in the endpoints of their world lines and their Doppler-shifted frequencies. The polarizations of both photons will be measured, but one measurement (the encounter event) will be the cause of the collapse. At the other end of the symmetric interval the polarization state of the other photon collapses, the collapse event. The collapse event always occurs prior to the detection of the other particle, the response event.

A. Invariant Form

As we are interested in the slope of the symmetric interval, and since there is no reason to presume this slope is dependent upon the absolute frequencies of the photons, it must depend only on their relative frequencies. Since there is no preferred reference frame the functional form of this dependence must be the same in every frame. Let β be the slope in the lab frame, β' the slope in an arbitrary frame, and α the ratio of frequencies, then in functional form $f(\alpha)$:

$$\begin{aligned} \beta &= f\left(\frac{\nu_2}{\nu_1}\right) = f(\alpha) \\ \beta' &= f\left(\frac{\nu_2'}{\nu_1'}\right) = f(\alpha') \end{aligned} \tag{1}$$

Given the encounter $\{x_e,t_e\}$ and collapse events $\{x_c,t_c\}$, the slope in an arbitrary reference frame must be:

$$\beta' = \frac{t_e' - t_c'}{x_e' - x_c'} \tag{2}$$

and performing the Lorentz transformation for an observer moving at velocity u, (natural units):

$$\beta' = \frac{\gamma(t_e - ux_e) - \gamma(t_c - ux_c)}{\gamma(x_e - ut_e) - \gamma(x_c - ut_c)} \tag{3}$$

yields:

$$\beta' = \frac{\beta - u}{1 - u\beta} \tag{4}$$

The relationship between the ratio of frequencies in the two frames is given by the Doppler shift:

$$\alpha' = \frac{1-u}{1+u}\alpha \tag{5}$$

Presuming that $f(\alpha)$ relates to a physical variable, it is also presumed analytic and in particular continuous in the limit of slow relativistic frames;

$$\begin{aligned} &u \rightarrow \Delta u \approx 0 \\ &\alpha'(\Delta u) = \alpha + \Delta\alpha \end{aligned} \tag{6}$$

Substituting equation (4) into equation (5) yields;

$$\begin{aligned} &\alpha' \approx (1 - 2\Delta u)\alpha \\ &\Delta\alpha = -2\alpha\Delta u \end{aligned} \tag{7}$$

The change in the slope of the symmetric interval can be found from a Taylor series expansion of equation (1) and substituting equation (5) yields;

$$\beta' = \beta - \frac{d\beta}{d\alpha}2\alpha\Delta u \tag{8}$$

Since $u = \Delta u$ is small, equation (4) for β' yields;

$$\beta' \approx \beta - \Delta u + \Delta u\beta^2 \tag{9}$$

Since these two expressions for β' must be the same;

$$\beta - \frac{d\beta}{d\alpha} 2\alpha\Delta u = \beta - \Delta u + \Delta u \beta^2 \tag{10}$$

which yields the differential equation;

$$\frac{d\beta}{d\alpha} = \frac{1-\beta^2}{2\alpha} \tag{11}$$

Integrating and noticing that β is always less than one and $\alpha = 1$ when $\beta = 0$;

$$\alpha = \frac{1+\beta}{1-\beta} \tag{12}$$

Solving for β, but in steps for later use, yields;

$$\nu_1(1+\beta) = \nu_2(1-\beta) \tag{13}$$

continuing;

$$\beta = \frac{\nu_2 - \nu_1}{\nu_2 + \nu_1} \tag{14}$$

and substituting into equation (4);

$$\beta' = \frac{\frac{\nu_2 - \nu_1}{\nu_2 + \nu_1} - u}{1 - u\frac{\nu_2 - \nu_1}{\nu_2 + \nu_1}} \tag{15}$$

Expressing in terms of the Doppler shifted frequencies yields;

$$\beta' = \frac{\nu_2' - \nu_1'}{\nu_2' + \nu_1'} \tag{16}$$

which is the same functional form as for β.

B. Physical Significance

Now what physical significance can be associated with β? Consider equation (13) and convert to its representation in momentum units;

$$\frac{h\nu_1}{c}(1+\beta) = \frac{h\nu_2}{c}(1-\beta) \tag{17}$$

If we now regard β as a velocity;

$$\frac{h\nu_1'}{c} = \frac{h\nu_2'}{c} \tag{18}$$

then it represents the velocity of the center of mass of the entangled particles. Therefore, the slope of the symmetric spacetime interval is equal to the velocity of the center of mass of the entangled system. We will take this as a general principle of physics.

C. Generalizing the Hypothesis

For a system of two entangled particles with detectors to observe each, we can now specify which measurement event is the cause and which the effect. In the center of mass frame, it is the first measurement. However, this way of thinking about it makes it look like there is a preferred frame when in reality there isn't. In order to see this, it is convenient to introduce a bit of jargon.

Consider Figure 3, a spacetime diagram for two entangled sublight particles. The light cone consists of the two dashed lines at 45°. The nearly vertical world lines are for the two detectors, D1 and D2. Particle one moves to the left (world line a), and particle two moves to the right (world line b). Their center of mass is shown as the dotted world line m, and the symmetric interval is the fat line in gray (spacelike interval s). The detection of the encounter particle (on the right) causes the collapse of the state of the responder particle prior to its measurement. All observers can agree on which detection is the cause and which the effect, even though some of them (including this one) see the collapse as proceeding backward in time. The encounter (e), collapse (c), & responder (r) spacetime events are shown.

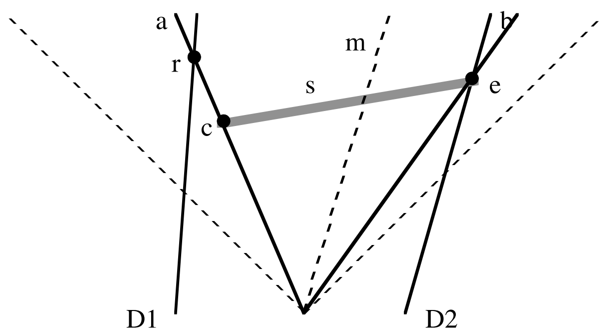

Figure 3: Two entangled matter particles connected by a symmetric spacetime interval (gray line) with slope equal to the velocity of their center of mass (dotted line).

To determine which measurement is the encounter event, simply draw two spacelike intervals with slope equal to the velocity of the center of mass, one through each measurement event. Since they will be parallel and spacelike, one will be everywhere earlier in time than the other. The interval earlier in time (and it will be earlier in every reference frame) is the symmetric interval.

Given the world line of the encounter detector, the collapse event can be computed from the encounter event. Let the origin of the reference frame be the formation of the entangled particles, with the encounter detector initially at a distance D, moving with velocity, v_d. Intersecting the world lines of the encounter detector and the encounter particle;

$$\{x_e, t_e\} = \left\{ \frac{v_e D}{v_e - v_d}, \frac{D}{v_e - v_d} \right\} \tag{19}$$

Therefore, the equation for the line of the symmetric interval can be found from its slope and its intersection with the world line of the responder particle;

$$t = \beta x + t_e(1 - \beta v_e) \tag{20}$$

and its intersection with the world line of the responder particle yields the collapse event;

$$\{x_c, t_c\} = \left\{ v_r \frac{t_e(1 - \beta v_e)}{1 - \beta v_r}, \frac{t_e(1 - \beta v_e)}{1 - \beta v_r} \right\} \tag{21}$$

As can be seen, collapse is indeed instantaneous in the center of mass frame ($\beta = 0$), i.e., $t_c = t_e$, and depending on the sign and magnitude of β, the collapse can be propagating either forward or backward in time.

V. Conclusion

Since the world line for a quantum object is "thick", when a position measurement is made on it, its world line suffers a discontinuous thinning, leaving an edge which must be spacelike. It was hypothesized in the previous paper that this edge would be a symmetric spacetime interval. In order to derive the slope of this interval, the simple situation of two entangled photons was considered. Since the slope of the interval can depend only on the frequencies of the photons, its mathematical form must be the same in every frame. The mathematical form was derived and shown to be frame independent. It turned out that physical significance could be associated with the slope of the symmetric interval; it was equal to the velocity of the center of mass of the entangled photons. This result was generalized to include matter particles as a formal hypothesis.

Acknowledgments

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